One-step generation of high-quality squeezed and EPR states in cavity QED.

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Abstract

We show how to generate bilinear (quadratic) Hamiltonians in cavity quantum electrodynamics (QED) through the interaction of a single driven three-level atom with two (one) cavity modes. With this scheme it is possible to generate one-mode mesoscopic squeezed superpositions, two-mode entanglements, and two-mode squeezed vacuum states (such the original EPR state), without the need for Ramsey zones and external parametric amplification. The degree of squeezing achieved is up to 99% with currently feasible experimental parameters and the errors due to dissipative mechanisms become practically negligible.

The preparation of Einstein-Podolsky-Rosen (EPR) entanglement [1] has been a challenge for theoretical and experimental physics since the introduction of the Bell inequalities [2] to test for nonlocal correlations. Over the last few decades a plethora of experimental confirmations of nonlocal correlation has contributed to widening the perspectives for applications of this fundamental phenomenon, from quantum teleportation [3] to computation [4]. Equally impressive are the applications proposed for squeezed states, ranging from fundamental physics to technology. Possible ways of measuring gravitational waves through squeezed fields [5] and of deepening our understanding of the properties of radiation [6] and its interaction with matter [7] have been pursued alongside the preparation of such nonclassical states.

The preparation of squeezed light, supplied by nonlinear optical media as running waves [8] and standing squeezed fields in high-Q cavities and ion traps, generated through atom-field interaction [9], has already been investigated. The issue of squeezing an arbitrary previously-prepared cavity-field state $|\Psi\rangle$ had not been addressed until Refs. [10, 11]. Working with Rydberg atoms in the microwave regime, in this letter,we present a feasible enhanced scheme to engineer bilinear Hamiltonians in cavity QED. As in Refs. [10, 11], this is accomplished through the interactions of a single three-level atom simultaneously with a classical driving field and a two-mode cavity. However, the new scheme exhibits a crucial difference from the two previous ones [10, 11], both of which operate only in the weak-amplification regime, owing to the adiabatic approximation required. Here, both the parametric up- and down-conversions (PUC and PDC) are also derived for the strong-amplification regime, where the strength of the bilinear and quadratic interactions between the cavity modes are considerably increased. Hence, a smaller atom-field interaction time is required for state engineering purposes and, consequently, the atom-field dissipative mechanisms become negligible. These interactions are used to generate superpositions of highly-squeezed states, two-mode squeezed vacuum states (such as the EPR state), the even and odd EPR states, and entanglements of coherent states. Such states, generated without the need for Ramsey zones and external parametric amplification, can be employed either for fundamental tests of quantum mechanics or to manipulate quantum information.

PDC. Consider the atomic levels in the ladder configuration, as shown in Fig. 1a. The ground $|g\rangle$ and excited $|e\rangle$ states are coupled through an intermediate level $|i\rangle$. The cavity modes ω_a and ω_b ($\omega_a + \omega_b = 2\omega_0$) are tuned to the vicinity of the dipole-allowed transitions

$|g\rangle \leftrightarrow |i\rangle$ and $|e\rangle \leftrightarrow |i\rangle$ with coupling constants λ_a and λ_b, respectively, and detuning $\Delta = \omega_0 + \omega_i - \omega_a = -(\omega_0 - \omega_i - \omega_b)$. The desired interaction between the modes ω_a and ω_b is accomplished by driving out of resonance the dipole-forbidden atomic transition $|g\rangle \leftrightarrow |e\rangle$ [12] with a classical field of frequency $\omega = 2(\omega_0 - \delta)$ and coupling constant Ω. Within the rotating-wave approximation, the Hamiltonian is given by $H = H_0 + V(t)$, where (with $\hbar = 1$)

$$H_0 = \omega_a a^\dagger a + \omega_b b^\dagger b + \omega_0 (\sigma_{ee} - \sigma_{gg}) + \omega_i \sigma_{ii}, \tag{1a}$$

$$V(t) = \left(\lambda_a a \sigma_{ig} + \lambda_b b \sigma_{ei} + \Omega\, \mathrm{e}^{-i\omega t}\, \sigma_{eg} + \text{h.c.}\right). \tag{1b}$$

with $a^\dagger$ (a) and $b^\dagger$ (b) standing for the creation (annihilation) operators of the quantized cavity modes, while $\sigma_{kl} \equiv |k\rangle \langle l|$ (k, l being the atomic states). In a frame rotating with the driving-field frequency, obtained through the unitary operator $U = \exp\left[-i\omega t \left(a^\dagger a + b^\dagger b + \sigma_{ee} - \sigma_{gg}\right)/2\right]$, the transformed Hamiltonian $\widetilde{H} = \widetilde{H}_0 + \widetilde{V}$ is given by $\widetilde{H}_0 = \delta_a a^\dagger a + \delta_b b^\dagger b + \delta(\sigma_{ee} - \sigma_{gg}) + \omega_i \sigma_{ii} + (\Omega \sigma_{eg} + \text{h.c.})$ and $\widetilde{V} = (\lambda_a a \sigma_{ig} + \lambda_b b \sigma_{ei} + \text{h.c.})$, where $\delta_\ell = \omega_\ell - \omega/2$ ($\ell = a, b$). Assuming that $\delta \ll |\Omega|$ and defining a new basis for the atomic states $\left\{|i\rangle, |\pm\rangle = \left(\pm\, \mathrm{e}^{i\varphi/2} |g\rangle + \mathrm{e}^{-i\varphi/2} |e\rangle\right)/\sqrt{2}\right\}$[13], composed of eigenstates of the free atomic Hamiltonian, we obtain in the interaction picture

$$\begin{aligned}\mathcal{H}(t) = {} & \lambda_a\, \mathrm{e}^{i\varphi/2}\, a \left(\mathrm{e}^{i(\Delta - |\Omega| - \delta)t}\, \sigma_{i+} + \mathrm{e}^{i(\Delta + |\Omega| - \delta)t}\, \sigma_{i-}\right)/\sqrt{2} \\ & + \lambda_b\, \mathrm{e}^{i\varphi/2}\, b \left(\mathrm{e}^{-i(\Delta - |\Omega| + \delta)t}\, \sigma_{+i} + \mathrm{e}^{-i(\Delta + |\Omega| + \delta)t}\, \sigma_{-i}\right)/\sqrt{2} + \text{h.c.}\end{aligned} \tag{2}$$

where $\Omega = |\Omega|\, \mathrm{e}^{-i\varphi}$. In what follows we discuss two regimes of the classical amplification field: the weak ($|\lambda_a|, |\lambda_b| < |\Omega| \ll \Delta$) and the strong ($|\Omega| \gg \Delta, |\lambda_a|, |\lambda_b|$) amplification regimes. In both cases, the Hamiltonian (2) consists of highly oscillating terms and to a good approximation we finally obtain the Hamiltonian $\mathcal{H}_{eff}(t) = -i\mathcal{H}(t) \int \mathcal{H}(\tau) d\tau$ [14], given by

$$\begin{aligned}\mathcal{H}_{eff}(t) \simeq {} & \frac{1}{\Delta^2 - |\Omega|^2} \left[\Delta |\lambda_a|^2 a a^\dagger + \Delta |\lambda_b|^2 b^\dagger b - |\Omega| \left(\lambda_a \lambda_b\, \mathrm{e}^{-2i\delta t}\, ab + \text{h.c.}\right)\right] \sigma_{ii} \\ & - \frac{1}{2(\Delta - |\Omega|)} \left[|\lambda_a|^2 a^\dagger a + |\lambda_b|^2 b b^\dagger + \left(\lambda_a \lambda_b\, \mathrm{e}^{-2i\delta t}\, ab + \text{h.c.}\right)\right] \sigma_{++} \\ & - \frac{1}{2(\Delta + |\Omega|)} \left[|\lambda_a|^2 a^\dagger a + |\lambda_b|^2 b b^\dagger - \left(\lambda_a \lambda_b\, \mathrm{e}^{-2i\delta t}\, ab + \text{h.c.}\right)\right] \sigma_{--}.\end{aligned} \tag{3}$$

PDC in the weak-amplification regime. To implement this regime it is sufficient to assume that $|\lambda_a| \sim |\lambda_b|$, $\Delta \gtrsim 10 \times |\lambda_a|$ and $|\Omega| \sim 2|\lambda_a|$. Starting with the atomic state prepared in subspace $\{|i\rangle\}$ or $\{|\pm\rangle\}$, and adjusting δ such that $\delta_i = -\left(|\lambda_a|^2 + |\lambda_b|^2\right)/\Delta$ or $\delta_\pm = \left(|\lambda_a|^2 + |\lambda_b|^2\right)/2\left(\Delta \mp |\Omega|\right) \approx \left(|\lambda_a|^2 + |\lambda_b|^2\right)/2\Delta$, we obtain from Eq. (3), after the unitary transformation $\exp\left\{-it\left(|\lambda_a|^2 a^\dagger a + |\lambda_b|^2 b^\dagger b\right)\left[\sigma_{ii}/\Delta - \sigma_{++}/2\left(\Delta - |\Omega|\right) - \sigma_{--}/2\left(\Delta + |\Omega|\right)\right]\right\}$, respectively

$$\mathcal{H}_i \simeq \left(\xi_i ab + \xi_i^* a^\dagger b^\dagger\right)\sigma_{ii}, \tag{4a}$$

$$\mathcal{H}_\pm \simeq \left(\xi_\pm ab + \xi_\pm^* a^\dagger b^\dagger\right)\left(\sigma_{--} - \sigma_{++}\right), \tag{4b}$$

where $\xi_i = \Omega^* \lambda_a \lambda_b / \Delta^2$ and $\xi_\pm = \lambda_a \lambda_b \, \mathrm{e}^{i\varphi}/2\left(\Delta \mp |\Omega|\right) \approx \lambda_a \lambda_b \, \mathrm{e}^{i\varphi}/2\Delta \equiv \xi$. We observe that Hamiltonian (4a) was used in Ref. [11] to generate squeezed states and the original Einstein-Podoslky-Rosen (EPR) entanglement, expanded in the Fock representation, in two-mode cavity QED. However, with the present technique, Hamiltonian (4b) has the advantage that the coupling parameter $\xi_\pm$ is at least one order of magnitude larger than ξ_i; consequently, the atom-field interaction time required to obtain a high-"quality" EPR state can be considerably shorter, making the dissipative effects negligible. Starting with the atom in the state $|\pm\rangle$, both cavity modes being in their vacuum states, and applying the interaction in Eq. (4b) during the time interval τ, the evolved two-mode state reads

$$\mathrm{e}^{-i\mathcal{H}_\pm \tau}\,|\pm\rangle\,|0,0\rangle_{ab} = |\pm\rangle \sum_{n=0}^{\infty} \frac{\left[\pm \tanh\left(|\xi|\,\tau\right)\right]^n}{\cosh\left(|\xi|\,\tau\right)}\,|n,n\rangle_{ab} = |\pm\rangle\,|\psi_\pm(\tau)\rangle_{ab}\,, \tag{5}$$

where we have adjusted the coupling constants λ_a, λ_b and classical phase φ so that $\xi = i\,|\xi|$. State $|\psi_+(\tau)\rangle_{ab}$ is the two-mode squeezed vacuum state which, in the limit $|\xi|\,\tau \to \infty$ (and projected into the positional basis of modes a and b), is exactly the original entanglement used in the EPR argument against the uncertainty principle [1].

To estimate the "quality" of the prepared EPR state $|\psi_+(\tau)\rangle_{ab}$ we compute the mean values [15] $(\Delta x)^2 = \left\langle (x_a - x_b)^2 \right\rangle = \mathrm{e}^{-2|\xi|\tau}/2$ and $(\Delta p)^2 = \left\langle (p_a + p_b)^2 \right\rangle = \mathrm{e}^{-2|\xi|\tau}/2$, where $x_\beta = \left(\beta + \beta^\dagger\right)/2$ and $p_\beta = -i(\beta - \beta^\dagger)/2$ $(\beta = a, b)$ are the field quadratures. We obtain the result $(\Delta x)^2 + (\Delta p)^2 = \mathrm{e}^{-2|\xi|\tau}$ which goes to zero for the ideal EPR state $(|\xi|\,\tau \to \infty)$ and to unity for an entirely separable state [15]. Therefore, the expression $1 - \mathrm{e}^{-2|\xi|\tau}$ can be used to estimate the quality of the prepared EPR state. Assuming typical values for the parameters involved in cavity QED experiments, we get $|\lambda_a| \sim |\lambda_b| \sim 3 \times 10^5 \mathrm{s}^{-1}$ [16]. With the detuning $\Delta \sim 10 \times |\lambda_a|$, it follows that $|\xi| \sim 1.5 \times 10^4 \mathrm{s}^{-1}$, and assuming the interaction

time $\tau \sim 5 \times 10^{-5}$s, we obtain $|\xi|\,\tau = 0.75$ which is larger than the value 0.69 achieved for building the EPR state for unconditional quantum teleportation in the running-wave domain [17]. The resulting quality of the prepared EPR state is $1 - \mathrm{e}^{-2|\xi|\tau} \sim 0.78$. (This should be compared with the EPR state engineered through Hamiltonian (4a) in Ref. [11], where the above cavity QED parameters lead to $|\xi_i| \sim 6 \times 10^3$ and the quality $1 - \mathrm{e}^{-2|\xi_i|\tau} \sim 0.45$.) The interaction time considered here is about three (one) orders of magnitude smaller than the field (atom) decay time in experiments employing closed microwave cavities [18], and about two (three) orders of magnitude smaller than the field (atom) decay time in experiments with open microwave cavities [16]. Consequently, the dissipative mechanism becomes negligible. We also note that the proposed scheme, where classical fields are necessary to induce a Raman transition (see [12]), is better suited for open cavities, although closed microwave cavities can be used as well (see discussion in [13]).

Besides having the advantage that $\xi \sim 5\ \xi_i$, this technique makes it is possible to generate mesoscopic superposition states, when preparing the atom, for example, in the excited state $|e\rangle = \mathrm{e}^{i\varphi/2}\left(|+\rangle + |-\rangle\right)/\sqrt{2}$. *i)* In the nondegenerate PDC, starting with both cavity modes in their vacuum states and applying the interaction in Eq. (4b) during the time interval τ, we generate the superposition of two-mode vacuum squeezed states $|\psi(\tau)\rangle_{ab} = \mathcal{N}_{\pm}\left(\mathrm{e}^{-i|\Omega|\tau}\,|\psi_{+}(\tau)\rangle_{ab}\,|+\rangle + \mathrm{e}^{i|\Omega|\tau}\,|\psi_{-}(\tau)\rangle_{ab}\,|-\rangle\right)$. Adjusting $|\Omega|\,\tau = 2\pi$ and measuring the atomic state after the atom-field interaction, we obtain the even and odd EPR states, which we define as

$$\left|\Psi_{\binom{even}{odd}}\right\rangle_{ab} = \mathcal{N}_{\pm}\left(|\psi_{+}(\tau)\rangle_{ab} \pm |\psi_{-}(\tau)\rangle\right) = \mathcal{N}_{\pm}\sum_{n=0}^{\infty}\left[1 \pm (-1)^{n}\right]\frac{\left[\tanh\left(|\xi|\,\tau\right)\right]^{n}}{\cosh\left(|\xi|\,\tau\right)}\,|n,n\rangle_{ab}\,. \qquad (6)$$

Similarly to the even and odd coherent states [19], $\langle\psi_{\pm}(\tau)\,|\psi_{\mp}(\tau)\rangle = \cosh^{-1}(2\,|\xi|\,\tau) \sim 2\,\mathrm{e}^{-2|\xi|\tau}$ for $2\,|\xi|\,\tau \gg 1$, while $\langle\Psi_{even}\,|\Psi_{odd}\rangle_{ab} = 0$. *ii)* In the degenerate PDC ($\omega_a = \omega_b$), the engineered cavity-field superposition, after the atom-field interaction and the measurement of the atomic state, is written as $|\Phi\,(\tau)\rangle = \mathcal{N}_{\pm}\left[\mathrm{e}^{-i|\Omega|\tau}\,S(\xi,\tau) \pm \mathrm{e}^{i\Omega\tau}\,S^{-1}(\xi,\tau)\right]|\Phi\,(0)\rangle$, where $S(\xi,\tau) = \exp\left[-i\left(\xi a^{\dagger 2} + \xi^{*}a^{2}\right)\tau\right]$ stands for the squeeze operator (with the squeezing factor $r = 2\,|\xi|\,\tau$), and the $+$ ($-$) sign occurs if the atom is detected in state $|e\rangle$ ($|g\rangle$). We note that the components of the superposition $|\Phi\,(\tau)\rangle$ are squeezed in perpendicular directions. Besides the large coupling strength $|\xi|$ achieved, another advantage of the present scheme to generate the superpositions in Eq. (6) and $|\Phi\,(\tau)\rangle$ is that no Ramsey zones or external parametric amplification are required. We also note that the degenerate PDC can be

implemented considering non-circular Rydberg states in an appropriated configuration as discussed in Ref. [20].

To estimate the degree of squeezing achieved with the present scheme we consider the degenerate PDC, starting with the cavity mode prepared in a coherent state $|\alpha\rangle$ and the atom prepared in the state $|+\rangle$. The variance in the squeezed quadrature of the generated coherent squeezed state $S(\xi,\tau)\,|\alpha\rangle$ is $\langle\Delta X\rangle^2 = \mathrm{e}^{-4|\xi|\tau}/4$. Assuming the typical cavity-QED values defined above for $|\lambda_a|$, $|\lambda_b|$, Δ, and τ, we finally obtain the squeezing factor $r = 2\,|\xi|\,\tau = 1.5$. With such values, the variance in the squeezed quadrature turns out to be $\langle\Delta X\rangle^2 \sim 1.2\times 10^{-2}$, representing a squeezing around 95%.

PDC in the strong-amplification regime. Here we assume that $|\lambda_a| \sim |\lambda_b| \gtrsim \Delta$ and $|\Omega| \gtrsim 10\times|\lambda_a|$. For the atomic state prepared in subspace $\{|i\rangle\}$ or $\{|\pm\rangle\}$, and assuming $\delta = 0$, we obtain from (3) the Hamiltonians

$$\mathcal{H}_i \simeq -\left(\zeta_i ab + \zeta_i^* a^\dagger b^\dagger\right)\sigma_{ii}, \tag{7a}$$

$$\mathcal{H}_\pm \simeq \frac{1}{2\,|\Omega|}\left(|\lambda_a|^2\,a^\dagger a + |\lambda_b|^2\,bb^\dagger\right)(\sigma_{++}-\sigma_{--}) + \left(\zeta_\pm ab + \zeta_\pm^* a^\dagger b^\dagger\right)(\sigma_{++}+\sigma_{--})\,, \tag{7b}$$

where $\zeta_i = \lambda_a\lambda_b\,\mathrm{e}^{i\varphi}\,/\,|\Omega|$ and $\zeta_\pm = \lambda_a\lambda_b/2\,(|\Omega|\mp\Delta) \approx \lambda_a\lambda_b/2\,|\Omega| \equiv \zeta$. Differently from the weak-amplification regime, here we may assume $\Delta = 0$. Note that apart from the global phase, Hamiltonian (7a) is exactly the same as (4a), but with the advantage that $\zeta_i \sim 10\times\xi_i$. With the cavity-QED experimental parameters defined above, we obtain $|\zeta_i|\,\tau = 1.5$, so that the EPR state generated through Hamiltonian (7a) has the quality $1-\mathrm{e}^{-2|\zeta_i|\tau} \sim 0.95$. Hence, in the degenerate PDC, the squeezing factor obtained when engineering a squeezed coherent state from Hamiltonian (7a) is $2\,|\zeta_i|\,\tau = 3.0$ and the variance in the squeezed quadrature becomes $\langle\Delta X\rangle^2 \sim 6.2\times 10^{-4}$, leading to a squeezing of 99.7%.

Hamiltonian (7b) also possesses considerable advantages, compared to Eq. (4a), even though $\zeta \sim \xi$ (for the parameter values assumed so far). Considering again the degenerate PDC process and the atom prepared in the excited state $|e\rangle = \mathrm{e}^{i\varphi/2}\,(|+\rangle + |-\rangle)/\sqrt{2}$, the resulting Hamiltonian, leading to two uncoupled evolutions for the atom-field system, depending on the atomic state $|+\rangle$ or $|-\rangle$, is written in the Schrödinger picture as $\mathcal{H}_\pm = (\omega\pm\chi)\,a^\dagger a + \left(\zeta\,\mathrm{e}^{-2i\omega t}\left(a^\dagger\right)^2 + \zeta^*\,\mathrm{e}^{2i\omega t}\,a^2\right)$, with $\chi = \left(|\lambda_a|^2 + |\lambda_b|^2\right)/2\,|\Omega|$ $(\sim 2\,|\zeta|)$. This Hamiltonian was analyzed in detail in Ref. [22], using the time-dependent invariants by Lewis and Riesenfeld [21] and for the values presented above it corresponds to the case of critical coupling, where $|\zeta|/\chi = 1$. Starting from the initial state

$\mathrm{e}^{i\varphi/2}\left(|+\rangle + |-\rangle\right)|\alpha\rangle/\sqrt{2}$, $|\alpha\rangle$ being a coherent state injected into the cavity, the evolved superposition reads $\left(|+\rangle U_+ + |-\rangle U_-\right)|\alpha\rangle/\sqrt{2}$, where $U_\pm$ stands for the evolution operator associated with Hamiltonian $\mathcal{H}_\pm$, as derived in Ref. [22]. After interacting with the cavity mode, during the time interval τ, the atomic state is measured and we finally obtain the cavity-field superposition

$$|\Psi(\tau)\rangle = \mathcal{N}_\pm \left(\mathrm{e}^{-i|\Omega|\tau} U_+ \pm \mathrm{e}^{i|\Omega|\tau} U_-\right)|\alpha\rangle\,, \tag{8}$$

where the sign $+(-)$ arises from the detection of the state $|e\rangle$ ($|g\rangle$). The interesting feature of this scheme for engineering a squeezed "Schrödinger cat"-like state is that no Ramsey zones or external parametric amplification field are needed, differently from Ref. [22]. The squeezing factor achieved, $r = \operatorname{arcsinh}(2\,|\zeta|\,\tau)$, computed in Ref. [22], is around 1.2, assuming typical parameters in cavity QED. It has been shown in Ref.[22] that the decoherence time of such mesoscopic superposition could be around the relaxation time of the cavity field when assuming a particular squeezed reservoir at absolute zero. This reservoir must be composed of oscillators squeezed in a direction perpendicular to that of the superposition state. Therefore, in this paper we have solved partially the problem of engineering a truly mesoscopic state in cavity QED. The task of engineering an optimal squeezed reservoir in cavity QED remains to be achieved. In Ref. [23] the authors show how to prepared a partially-squeezed reservoir in cavity QED.

PUC. Now, the energy diagram of the Rydberg three-level atom is in the Λ configuration, where the ground and excited states are coupled through an auxiliary more-excited level, as in Fig. 1b. The cavity microwave modes of frequencies ω_a and ω_b ($\omega_a - \omega_b = 2\omega_0$) enable both dipole-allowed transitions $|g\rangle \leftrightarrow |i\rangle$ and $|e\rangle \leftrightarrow |i\rangle$, with coupling constants λ_a and λ_b, respectively, and detuning $\Delta = \omega_i + \omega_0 - \omega_a = \omega_i - \omega_0 - \omega_b$. (Evidently, in this case we can assume at most two levels as circular Rydberg states.) A classical field of frequency $\omega = 2\left(\omega_0 - \delta\right)$, driving the atomic transition $|g\rangle \leftrightarrow |e\rangle$ with coupling constant Ω, leads to the desired interaction between the modes ω_a and ω_b. Within the rotating wave approximation, the Hamiltonian $H = H_0 + V(t)$, is given by

$$H_0 = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b + \hbar\omega_g\sigma_{gg} + \hbar\omega_e\sigma_{ee} + \hbar\omega_i\sigma_{ii}, \tag{9a}$$

$$V(t) = \hbar\left(\lambda_a a\sigma_{ig} + \lambda_b b\sigma_{ie} + \Omega e^{-i\omega t}\sigma_{ge} + \text{h.c.}\right). \tag{9b}$$

Following analogous steps to those used in the above analysis for the PDC, we obtain the effective Hamiltonian for the PUC.

PUC in the weak-amplification regime. From an atomic state prepared in subspace $\{|i\rangle\}$ or $\{|\pm\rangle\}$, adjusting δ as $\delta_i = \left(|\lambda_b|^2 - |\lambda_a|^2\right)/2\Delta$ or $\delta_\pm = \left(|\lambda_a|^2 - |\lambda_b|^2\right)/4\left(\Delta \mp |\Omega|\right) \approx \left(|\lambda_a|^2 - |\lambda_b|^2\right)/4\Delta$, we obtain, respectively

$$\mathcal{H}_i \simeq \left(\gamma_i a b^\dagger + \gamma_i^* a^\dagger b\right) |i\rangle \langle i| \,, \tag{10a}$$

$$\mathcal{H}_\pm \simeq \left(\gamma_\pm a b^\dagger + \gamma_\pm^* a^\dagger b\right) \left(|-\rangle \langle -| - |+\rangle \langle +|\right), \tag{10b}$$

where $\gamma_i = |\Omega| \lambda_a \lambda_b^* \mathrm{e}^{i\varphi} / \Delta^2$ and $\gamma_\pm = \lambda_a \lambda_b^* \mathrm{e}^{i\varphi} / 2\left(\Delta \mp |\Omega|\right) \approx \lambda_a \lambda_b^* \mathrm{e}^{i\varphi} / 2\Delta \equiv \gamma$. After preparing the atom in the ground state and mode ω_a (ω_b) in the coherent state $|\alpha\rangle_a$ ($|\beta\rangle_b$), the interaction (10b) can be used to generate cavity-field entangled states (without the need for Ramsey zones) such as $|\psi(\tau)\rangle_{ab} = \left(\mathrm{e}^{|\gamma|\tau\left(ab^\dagger - a^\dagger b\right)} |+\rangle + \mathrm{e}^{-|\gamma|\tau\left(ab^\dagger - a^\dagger b\right)} |-\rangle\right) |\alpha, \beta\rangle_{ab} / \sqrt{2}$, where we have assumed $\gamma = -i|\gamma|$. Adjusting the atom-field interaction time so that $|\gamma|\tau = \pi/2$, we obtain, after the atomic-state detection, $|\psi_\pm(\tau)\rangle_{ab} = \mathcal{N}_\pm \left(\mathrm{e}^{-i|\Omega|\tau} |\beta, -\alpha\rangle \pm \mathrm{e}^{i|\Omega|\tau} |-\beta, \alpha\rangle\right)$, where the $+$ $(-)$ sign occurs if the atom is detected in state $|e\rangle$ ($|g\rangle$).

PUC in the strong-amplification regime. In this case, we obtain exactly the Hamiltonians in Eqs. (10a) and (10b), but with coupling strengths $\eta_i = \lambda_a \lambda_b^* \mathrm{e}^{i\varphi} / |\Omega|$ and $\eta_\pm = \mp \lambda_a \lambda_b^* \mathrm{e}^{i\varphi} / 2|\Omega|$. Since η_i is around an order of magnitude larger than γ_i, beam-splitter operations can be realized in the strong-amplification regime with a smaller atom-field interaction time.

Finally, we discuss some possible error sources in our scheme, starting with the validaty of the effective Hamiltonians coming from the approximation $\mathcal{H}_{eff}(t) = -i\mathcal{H}(t) \int \mathcal{H}(\tau) d\tau$. In order to compare the evolutions governed by the effective Hamiltonians $\mathcal{H}_i$ and $\mathcal{H}_\pm$, for the PUC, with those calculated without this approximation, defined in Eqs (9a) and (9b), we computed numerically the time evolution of the initial states $|i\rangle |1\rangle_a |0\rangle_b$ and $|\pm\rangle |1\rangle_a |0\rangle_b$ in both cases. For the experimental parameters defined above, the divergence between the curves is about 5% in both regimes of amplification. Evidently, when considering a ratio $\Delta / |\lambda_a| > 10$ ($|\Omega| / |\lambda_a| > 10$) in the weak (strong) coupling regime, the approximation becomes even better, at the expense of a larger atom-field interaction time.

Focusing on the squeezed coherent state engineered in the degenerate PDC in the strong-amplification regime, we have estimated the squeezing factor $\widetilde{r} = 2|\zeta_i| (1 - \mathrm{e}^{-\Gamma_a \tau})/\Gamma_a$ and the variance of the squeezed quadrature $\left\langle \Delta \widetilde{X} \right\rangle^2 = \left[1 - \left(1 - \mathrm{e}^{-2\widetilde{r}}\right) \mathrm{e}^{-\Gamma_c \tau}\right]/4$ taking into account the cavity damping rate (Γ_c) and the atomic decay (Γ_a), which are introduced phenomenologically following the reasoning in Ref. [10]. Considering non-circular Rydberg

states in experiments with open cavities, such that $\Gamma_c \sim 10^3$ s^{-1} [16] and $\Gamma_a \sim 5 \times 10^3$ s^{-1}, together with the values assumed above and an atom-field interaction time about $\tau \sim 5 \times 10^{-5}$s, we obtain $\widetilde{r} \sim 2.6$ and $\langle \Delta X \rangle^2 \sim 1.3 \times 10^{-2}$, representing a squeezing around 95%. As mentioned above, for the atom-field interaction time required in our technique, the dissipative mechanism becomes practically negligible. We finally mention that a sample of N identical atoms may be employed to enhance the squeezing factor as recently considered by Guzman et al. [24].

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[12] This dipole-forbidden transition can be induced by taking advantage of dipole-allowed transitions to another auxiliary level. In this case, a fourth level $|f\rangle$ must be added to the atomic system together with appropriated classical fields inducing a Raman transition with effective coupling given by $\Omega = g_1^* g_2/\widetilde{\Delta}$, g_1 and g_2 being the coupling constants to the vicinity of the

dipole-allowed transitions $|g\rangle \leftrightarrow |f\rangle$ and $|e\rangle \leftrightarrow |f\rangle$, both with detuning $\widetilde{\Delta} \gg |g_1|,|g_2|$. For the strong coupling regime it is required that $\Omega \sim 10^6\mathrm{s}^{-1}$, which follows from $g_1 \sim g_2 \sim 10^7\mathrm{s}^{-1}$ (easily achieved for dippole-allowed transitions). Note that we may also consider the auxiliary level $|i\rangle$ itself for this purpose, watching out for the classical fields to be far from resonance with the cavity modes.

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Fig. 1. Energy diagram of a three-level atomic system in the (a) ladder and (b) lambda configurations.1

Figure 1(a)

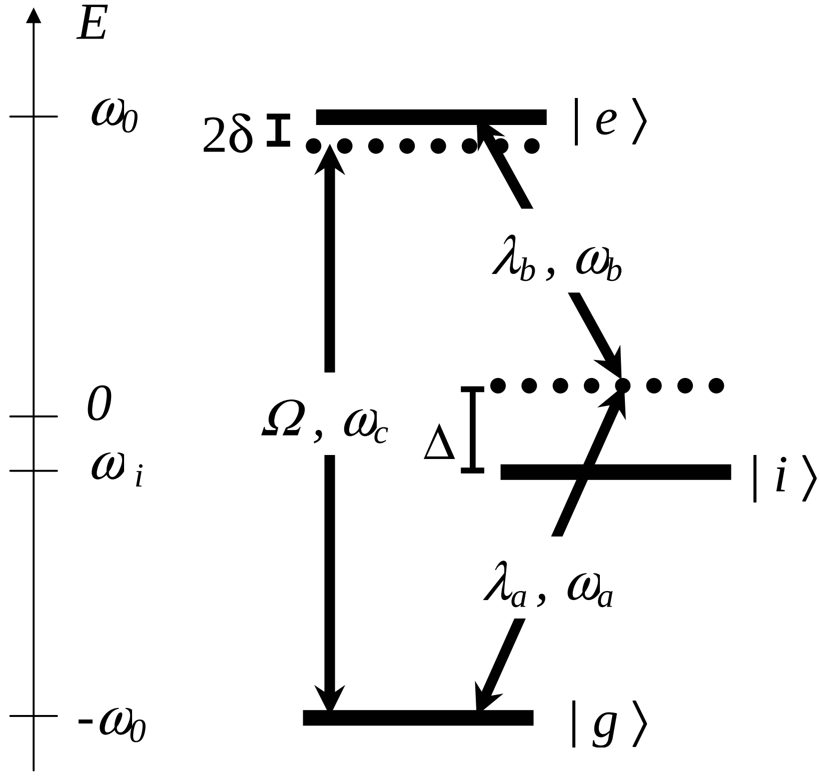

Figure 1(b)